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A New Fuzzy Interpolative Reasoning Method Based on Center of Gravity

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Abstract—**Interpolative reasoning methods do not only help reduce the complexity of fuzzy models but also make inference in sparse-rule based systems possible. This paper presents an interpolative reasoning method by exploiting the center of gravity (COG) property of the fuzzy sets concerned. The method works by first constructing a new inference rule via manipulating two given adjacent rules, and then by using similarity information to convert the intermediate inference results into the final derived conclusion. Two transformation operations are introduced to support such reasoning, which allow the COG of a fuzzy set to remain unaltered before and after the transformation. Results of esxperimental comparisons are provided to reflect the success of this work.**

I. INTRODUCTION

FUZZY rule interpolation helps reduce the complexity of fuzzy models and supports inference in systems that employ sparse rule sets [1]. With interpolation, fuzzy rules which may be approximated from their neighbouring rules can be omitted from the rule base. This leads to the complexity reduction of fuzzy models. When given observations have no overlap with the antecedent values of rules, classical fuzzy inference methods have no rule to fire, but interpolative reasoning methods can still obtain certain conclusions. Despite these significant advantages, earlier work in fuzzy interpolative reasoning does not guarantee the convexity of the derived fuzzy sets [3][4], which is often a crucial requirement of fuzzy reasoning to attain more easily interpretable practical results.

In order to eliminate the nonconvexity drawback, there has been considerable work reported in the literature. For instance, Vas, Kalmar and Kóczy have proposed an algorithm [6] that reduces the problem of nonconvex conclusions. Qiao, Mizumoto and Yan [7] have published an improved method which uses similarity transfer reasoning to guarantee the convex results. Hsiao, Chen and Lee [5] have introduced a new interpolative method which exploits the slopes of the fuzzy sets to obtain convex conclusions. General fuzzy interpolation and extrapolation techniques [8] and a modified α-cut based method [9] have also been proposed. In addition, Bouchon, Marsala and Rifqi have created an interpolative method based on graduality [10].

Nevertheless, the existing methods do not seem to make use of the center of gravity (COG) property of fuzzy sets, which is an essential feature that concurrently reflects the location and shape of the fuzzy sets concerned. This paper treats the COG as the core of any fuzzy membership function, and proposes two transformation operations which support the application of fuzzy interpolative reasoning, so that the reasoning results are guaranteed to be convex and normal.

The rest of the paper is organized as follows. Section II describes the relevant background of fuzzy interpolative techniques. Section III proposes the new interpolative reasoning method based on exploiting the COG property. Section IV gives examples to illustrate the use of this method. Finally, Section V concludes the paper and points out important further work.

II. BACKGROUND OF FUZZY RULE INTERPOLATIVE TECHNIQUES

Fuzzy rule interpolation [1][2], proposed first by Kóczy and Hirota, is an inference technique for fuzzy rule bases where the antecedents do not cover the whole input universe. Such techniques are essential for sparse rule-based fuzzy systems. The initial rule interpolation method, which is hereafter referred to as the KH algorithm for presentational simplicity, requires the following conditions to be satisfied: The involved fuzzy sets have to be of continuous, convex and normal membership functions, with bounded support.

An important notion in [1] is the "less than" relation between two fuzzy sets. Fuzzy set A_1 is said to be less than A_2, denoted by $A_1 \prec A_2$, if $\forall \alpha \in [0,1]$, the following conditions hold:

$$\inf\{A_{1\alpha}\} < \inf\{A_{2\alpha}\},\ \sup\{A_{1\alpha}\} < \sup\{A_{2\alpha}\}, \quad (1)$$

where $A_{1\alpha}$ and $A_{2\alpha}$ are respectively the α-cut of A_1 and that of A_2, $\inf\{A_{i\alpha}\}$ is the infimum of $A_{i\alpha}$, and $\sup\{A_{i\alpha}\}$ is the supremum of $A_{i\alpha}$, $i = 1, 2$.

For simplicity, suppose that two fuzzy rules are given:

$$If\ X\ is\ A_1\ then\ Y\ is\ B_1,$$
$$If\ X\ is\ A_2\ then\ Y\ is\ B_2,$$

which are briefly denoted as $A_1 \Rightarrow B_1$ and $A_2 \Rightarrow B_2$, respectively. Also, suppose that these two rules are adjacent, i.e., there is no any such rule existing that the antecedent value A of that rule is between the region of A_1 and A_2. To entail the interpolation in the region between the antecedent values of these two rules, i.e., to determine a new conclusion B^* when an observation A^* located between fuzzy sets A_1 and A_2 is given, it is commonly assumed, for convenience, that rules in a given rule base are arranged with respect to a partial ordering among the convex and normal fuzzy sets (CNF sets) of the antecedents variables. For the above two rules, this means that

$$A_1 \prec A^* \prec A_2. \quad (2)$$

The simplest interpolation which is linear can thus be written as:

$$\frac{d(A^*, A_1)}{d(A^*, A_2)} = \frac{d(B^*, B_1)}{d(B^*, B_2)}, \quad (3)$$

where $d(.,.)$ is typically the Euclidean distance between two fuzzy sets (though other distance metrics may be used as alternatives for this). This is illustrated in Fig. 1, where the lower and upper distances between α-cuts $A_{1\alpha}$ and $A_{2\alpha}$ are defined as follows:

$$d_L(A_{1\alpha}, A_{2\alpha}) = d(\inf\{A_{1\alpha}\}, \inf\{A_{2\alpha}\}), \quad (4)$$

$$d_U(A_{1\alpha}, A_{2\alpha}) = d(\sup\{A_{1\alpha}\}, \sup\{A_{2\alpha}\}). \quad (5)$$

From (4) and (5), (3) can be rewritten as:

$$\min\{B^*_\alpha\} = \frac{\frac{\inf\{B_{1\alpha}\}}{d_L(A^*_\alpha, A_{1\alpha})} + \frac{\inf\{B_{2\alpha}\}}{d_L(A^*_\alpha, A_{2\alpha})}}{\frac{1}{d_L(A^*_\alpha, A_{1\alpha})} + \frac{1}{d_L(A^*_\alpha, A_{2\alpha})}}, \quad (6)$$

$$\max\{B^*_\alpha\} = \frac{\frac{\sup\{B_{1\alpha}\}}{d_U(A^*_\alpha, A_{1\alpha})} + \frac{\sup\{B_{2\alpha}\}}{d_U(A^*_\alpha, A_{2\alpha})}}{\frac{1}{d_U(A^*_\alpha, A_{1\alpha})} + \frac{1}{d_U(A^*_\alpha, A_{2\alpha})}}. \quad (7)$$

Alternatively, let

$$\lambda_L = \frac{d_L(A^*_\alpha, A_{1\alpha})}{d_L(A_{2\alpha}, A_{1\alpha})}, \ \lambda_U = \frac{d_U(A^*_\alpha, A_{1\alpha})}{d_U(A_{2\alpha}, A_{1\alpha})}, \quad (8)$$

the same solution can then be obtained as follows:

$$\min\{B^*_\alpha\} = (1 - \lambda_L)\inf\{B_{1\alpha}\} + \lambda_L \inf\{B_{2\alpha}\}, \quad (9)$$

$$\max\{B^*_\alpha\} = (1 - \lambda_U)\sup\{B_{1\alpha}\} + \lambda_U \sup\{B_{2\alpha}\}. \quad (10)$$

From this, $B^*_\alpha = (\min\{B^*_\alpha\}, \max\{B^*_\alpha\})$ results. From B^*_α, the conclusion fuzzy set B^* can be constructed by the representation principle of fuzzy sets:

$$B^* = \bigcup_{\alpha \in [0,1]} \alpha B^*_\alpha. \quad (11)$$

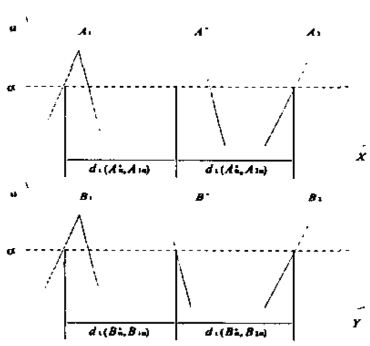

Fig. 1. Fuzzy interpolative reasoning with a nonconvex conclusion on a sparse fuzzy rule base

However, this linear interpolation cannot guarantee the convexity of the derived fuzzy sets (although they may be normal, as shown in Fig. 1), even when the fuzzy sets concerned in the given rules and the observations are all normal and convex.

III. THE PROPOSED METHOD

Center of gravity is an important property since it reflects both the location and the shape of a fuzzy set definition. Yet, it seems that none of the existing interpolative reasoning methods exploits this property. The present method takes the COG property into consideration to guide fuzzy interpolative reasoning in sparse rule bases.

For computational simplicity, it is herein presumed that all fuzzy sets used are triangular. Given a fuzzy set A as depicted in Fig. 2, with the three distinct coordinates of the triangular fuzzy set being $(a_0, 0)$, $(a_1, 1)$ and $(a_2, 0)$, the mathematical formula to calculate the COG of A are listed below:

$$COG(A)_x = \frac{a_0 + a_1 + a_2}{3}, \quad (12)$$

$$COG(A)_y = \frac{0 + 1 + 0}{3} = \frac{1}{3}. \quad (13)$$

Since $COG(A)_y$ is a constant, only the x-coordinate value is needed to be considered. The notation $COG(A)$ is therefore used to denote $COG(A)_x$, and the fuzzy set A is itself characterised by the triple (a_0, a_1, a_2) in the rest of this paper.

A. The Base Case

Suppose that two adjacent fuzzy rules $A_1 \Rightarrow B_1$, $A_2 \Rightarrow B_2$ and the observation A^*, which is located between fuzzy sets A_1 and A_2, are given. The general case of interpolative fuzzy reasoning concerning two variables X and Y can be described through the modus ponens interpretation below, and as illustrated in Fig. 3.

$$\begin{array}{l} \text{observation: } X \text{ is } A^* \\ \text{rules: if } X \text{ is } A_1, \text{ then } Y \text{ is } B_1 \\ \qquad\quad \text{if } X \text{ is } A_2, \text{ then } Y \text{ is } B_2 \\ \hline \text{conclusion: } Y \text{ is } B^*? \end{array} \quad (14)$$

Here, $A_i = (a_{i0}, a_{i1}, a_{i2})$, $B_i = (b_{i0}, b_{i1}, b_{i2})$, $i = 1, 2$, and $A^* = (a_0, a_1, a_2)$, $B^* = (b_0, b_1, b_2)$.

The proposed method begins with constructing a new fuzzy set A' which is close (or with a short distance) to *and* has the

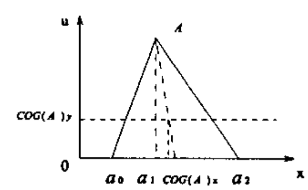

Fig. 2. Illustration of $COG(A)_x$ and $COG(A)_y$

same COG as A^*. To facilitate this, the distances defined in (4) and (5) are no longer employed. Instead, the distance between A_1 and A_2 is measured by that between $COG(A_1)$ and $COG(A_2)$:

$$d(A_1, A_2) = d(COG(A_1), COG(A_2)). \quad (15)$$

From this and by analogy to (8), in order to construct A', the following is computed:

$$\lambda_{COG} = \frac{d(A_1, A^*)}{d(A_1, A_2)} = \frac{d(COG(A_1), COG(A^*))}{d(COG(A_1), COG(A_2))}. \quad (16)$$

Then, a'_0, a'_1 and a'_2 of A' are calculated as follows:

$$a'_0 = (1 - \lambda_{COG})a_{10} + \lambda_{COG}a_{20}, \quad (17)$$

$$a'_1 = (1 - \lambda_{COG})a_{11} + \lambda_{COG}a_{21}, \quad (18)$$

$$a'_2 = (1 - \lambda_{COG})a_{12} + \lambda_{COG}a_{22}, \quad (19)$$

which are collectively abbreviated to

$$A' = (1 - \lambda_{COG})A_1 + \lambda_{COG}A_2. \quad (20)$$

Now, A' has the same COG as A^*, this is because

$$COG(A') = \frac{a'_0 + a'_1 + a'_2}{3}.$$

With (17)–(19) and (16),

$$\begin{aligned} & COG(A') \\ &= (1 - \lambda_{COG})\tfrac{a_{10}+a_{11}+a_{12}}{3} + \lambda_{COG}\tfrac{a_{20}+a_{21}+a_{22}}{3} \\ &= (1 - \lambda_{COG})COG(A_1) + \lambda_{COG}COG(A_2) \\ &= COG(A^*) \end{aligned}$$

From this, based on the essential idea of interpolation, a consequent fuzzy set B' can be obtained such that

$$b'_0 = (1 - \lambda_{COG})b_{10} + \lambda_{COG}b_{20}, \quad (21)$$

$$b'_1 = (1 - \lambda_{COG})b_{11} + \lambda_{COG}b_{21}, \quad (22)$$

$$b'_2 = (1 - \lambda_{COG})b_{12} + \lambda_{COG}b_{22}, \quad (23)$$

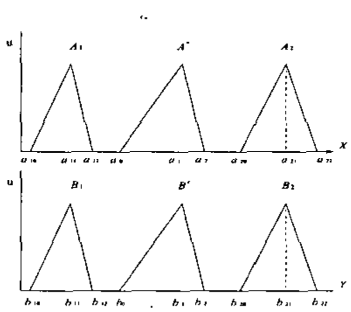

Fig. 3. Interpolation with triangular membership functions

with abbreviated notation:

$$B' = (1 - \lambda_{COG})B_1 + \lambda_{COG}B_2. \quad (24)$$

In so doing, the newly derived rule $A' \Rightarrow B'$ involves the use of only convex and normal fuzzy sets.

As $A' \Rightarrow B'$ is derived from $A_1 \Rightarrow B_1$ and $A_2 \Rightarrow B_2$, it is feasible to perform fuzzy reasoning with this new rule without further reference to its originals. The interpolative reasoning problem is therefore changed from (14) to the new modus ponens interpretation:

$$\begin{array}{l} \text{observation: } X \text{ is } A^* \\ \text{rule: if } X \text{ is } A', \text{ then } Y \text{ is } B' \\ \hline \text{conclusion: } Y \text{ is } B^*? \end{array} \quad (25)$$

This interpretation retains the same results as (14) in dealing with the extreme cases: If $A^* = A_1$, then from (16) $\lambda_{COG} = 0$, and according to (20) and (24), $A' = A_1$ and $B' = B_1$, so the conclusion $B^* = B_1$. Similarly, if $A^* = A_2$, then $B^* = B_2$.

Other than the extreme cases, *similarity* measures are used to support the application of this new modus ponens as done in [7]. In particular, (25) can be interpreted as

The more similar X to A', the more similar Y to B'. (26)

Suppose that a certain degree of similarity between A' and A^* is established, it is reasonable to require that the consequent parts B' and B^* attain this similarity degree. The question is now how to obtain an operator which will allow transforming B' to B^* with the desired degree of similarity. To this end, the following two component transformation operators are first introduced:

Scale Transformation Given a *scale rate* s, in order to transform the current fuzzy support $(a_2 - a_0)$ into a new support $(s * (a_2 - a_0))$ while keeping the COG and the ratio of left-support $(a'_1 - a'_0)$ to right-support $(a'_2 - a'_1)$ of the transformed fuzzy set the same as those of its original, that is, $COG(A') = COG(A)$ and $\frac{a'_1 - a'_0}{a'_2 - a'_1} = \frac{a_1 - a_0}{a_2 - a_1}$, the new a'_0, a'_1 and a'_2 must satisfy (as illustrated in Fig. 4. A):

$$a'_0 = \frac{a_0(1 + 2s) + a_1(1 - s) + a_2(1 - s)}{3}, \quad (27)$$

$$a'_1 = \frac{a_0(1 - s) + a_1(1 + 2s) + a_2(1 - s)}{3}, \quad (28)$$

$$a'_2 = \frac{a_0(1 - s) + a_1(1 - s) + a_2(1 + 2s)}{3}. \quad (29)$$

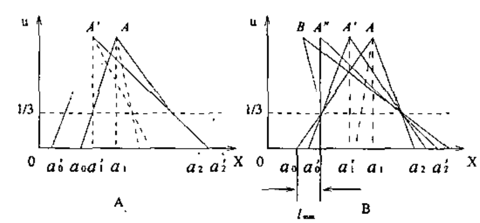

Fig. 4. Scale Transformation and Move Transformation

This is obvious. In fact, to satisfy the conditions imposed over the transformation, the linear equations below must hold simultaneously,

$$\begin{cases} a'_0 + a'_1 + a'_2 = a_0 + a_1 + a_2 \\ \frac{a'_1 - a'_0}{a'_2 - a'_1} = \frac{a_1 - a_0}{a_2 - a_1} \\ a'_2 - a'_0 = s(a_2 - a_0) \end{cases}$$

Solving these equations leads to the solutions as given in (27)–(29). Note that this scale transformation guarantees that the transformed fuzzy sets are convex thanks to the requirement of $\frac{a'_1 - a'_0}{a'_2 - a'_1} = \frac{a_1 - a_0}{a_2 - a_1}$.

Move Transformation : Given a move distance l, in order to transform the current fuzzy support $(a_2 - a_0)$ from the starting location a_0 to a new starting position $a_0 + l$ while keeping the COG and the length of support $(a_2 - a_0)$ remaining the same, i.e., $COG(A') = COG(A)$ and $a'_2 - a'_0 = a_2 - a_0$, the new a'_0, a'_1 and a'_2 must be (as shown in Fig. 4. B):

$$a'_0 = a_0 + l, \quad (30)$$
$$a'_1 = a_1 - 2l, \quad (31)$$
$$a'_2 = a_2 + l, \quad (32)$$

where $0 \le l \le l_{max} = (a_1 - a_0)/3$. If $l > l_{max}$, the transformation generates the nonconvex fuzzy sets. For instance, consider the extreme case where A is transformed to A'', as shown in Fig. 4. B. When $l = l_{max}$, the left slope of A'' becomes vertical. Any further increase in l will lead to the resulting transformed fuzzy set being a non CNF set. To avoid this, the *move rate* m is introduced:

$$m = \frac{l}{(a_1 - a_0)/3}. \quad (33)$$

If move rate $m \in [0, 1]$, then $l \le l_{max}$ holds. This ensures the transformed fuzzy set A' to be convex and normal if A is itself a CNF set. Note that the move transformation has two possible moving directions, the above discusses the left-direction case (from the viewpoint of a_1) with $l > 0$, the right direction with $l < 0$ should hold by symmetry:

$$m = \frac{l}{-(a_2 - a_1)/3} \in [0, 1]. \quad (34)$$

On top of the *scale* and *move transformations*, an *integrated transformation*, denoted as $T(A, A')$, between two fuzzy sets A and A' can be introduced such that A' is the derived CNF set of A by applying both transformation components. Obviously, two integrated transformations are said to be identical if and only if both of their *scale rate* and *move rate* are equal.

As indicated earlier, it is intuitive to maintain the similarity degree between the consequent parts B' and B^* to be the same as that between the antecedent parts A' and A^*, in performing interpolative reasoning. Now that the integrated transformation allows the similarity degree between two fuzzy sets to be measured by the *scale rate* and *move rate*, the desired conclusion B^* can be obtained by satisfying the following (as shown in Fig. 5):

$$T(B', B^*) = T(A', A^*). \quad (35)$$

That is, the parameters of *scale rate* and *move rate* calculated from A' to A^* are used to compute B^* from B' in a reverse engineering sense: Once B' is obtained, it is transformed with the same scale and move rates as what has been used to transform A^* to A', the result of this transformation is B^*. Clearly, B^* will then retain the same similarity degree as that between the antecedent parts A' and A^*.

There are two specific cases worth noting. The first is that if A^* is a singleton while A' is a CNF set, the scale transformation from A' to A^* is 0. This case can be easily handled by setting the result B^* to a singleton whose COG interpolates between $COG(B_1)$ and $COG(B_2)$ in the same way as A^* does between $COG(A_1)$ and $COG(A_2)$. The second case (which only exists if both antecedents A_1 and A_2 are singletons) is that if A^* is a CNF set while A' is a singleton, the scale transformation from A' to A^* is ∞. Since ∞ cannot be used to generate the resulting fuzzy set, a modified strategy is created for this. Let the *COG level width* of a fuzzy set A be the length between the two slopes at the α-cut level (which is $1/3$ for this case). The ratio between the *COG level width* of fuzzy set A^* and the distance of $COG(A_1)$ and $COG(A_2)$ is calculated, and then used to compute the *COG level width* of fuzzy set B^* by equalizing the corresponding ratio. Note that the fuzzy set obtained by the scale transformation from a singleton is an isosceles triangle. These two cases will be illustrated with examples later.

B. The General Case

The base case described in Section III.*A* concerns with interpolation between two adjacent rules with each involving one antecedent variable. However, the present approach is readily extendable to rules with multiple antecedent attributes.

Without losing generality, suppose that two adjacent rules R_i and R_j are represented by

$$if\ X_1\ is\ A_{1i}\ and \ldots and\ X_m\ is\ A_{mi}\ then\ Y\ is\ B_i,$$
$$if\ X_1\ is\ A_{1j}\ and \ldots and\ X_m\ is\ A_{mj}\ then\ Y\ is\ B_j.$$

Thus, when a vector of observations $(A_1^*, \ldots, A_k^*, \ldots, A_m^*)$ is given, by direct analogy to the base case, the values A_{ki} and

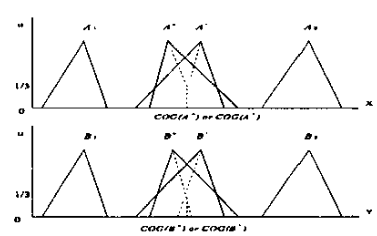

Fig. 5. Proposed interpolative method

A_{kj} of X_k, $k = 1, 2, \ldots, m$, are used to obtain the new CNF set A'_k:

$$A'_k = (1 - \lambda_k)A_{ki} + \lambda_k A_{kj}, \qquad (36)$$

where

$$\lambda_k = \frac{d(COG(A_{ki}), COG(A^*_k))}{d(COG(A_{ki}), COG(A_{kj}))}.$$

Clearly, the COG of A'_k will remain the same as that of the k-th observation A^*_k.

The resulting A'_k and the given A^*_k are used to compute the scale rate s_k and move rate m_k just like the one variable case. From this, the combined scale rate s_c and move rate m_c over the m conditional attributes are calculated as the arithmetic averages of s_k and m_k, $k = 1, 2, \ldots, m$:

$$s_c = \frac{1}{m}\sum_{k=1}^{m} s_k, \qquad (37)$$

$$m_c = \frac{1}{m}\sum_{k=1}^{m} m_k. \qquad (38)$$

Note that, other than using arithmetic average, different mechanisms such as the medium value operator may be employed for this purpose. However, the average helps to capture the intuition that when no particular information regarding which variable has a more dominating influence upon the conclusion, all the variables are treated equally. If such information is available, a weighted average operator may be better to use.

Regarding the consequent, by analogy to (24), B' can be computed by

$$B' = (1 - \lambda_a)B_i + \lambda_a B_j. \qquad (39)$$

Here, λ_a is deemed to be the average of λ_k, $k = 1, 2, \ldots, m$, to mirror the approach taken above

$$\lambda_a = \frac{1}{m}\sum_{k=1}^{m} \lambda_k. \qquad (40)$$

As the *combined scale rate* s_c and *move rate* m_c reflect the similarity degree between the observation vector and the values of the given rules, the fuzzy set B^* of the conclusion can then be estimated by transforming B' via the application of the same s_c and m_c.

IV. ILLUSTRATIVE EXAMPLES

In this section, the example problems given in [3][5] together with a new problem case are used to illustrate the newly proposed interpolative method and to facilitate comparative studies. All the results discussed below concern the interpolation between the two adjacent rules $A_1 \Rightarrow B_1$ and $A_2 \Rightarrow B_2$. In reporting these results, HCL stands for the work of [5] and HS stands for the work proposed in this paper.

Example 1. Now suppose $A^* = (7, 8, 9)$. Table I summarises the results, which are also depicted in Fig. 6. For this case, the

TABLE I

RESULTS FOR EXAMPLE 1, WITH $A^* = (7, 8, 9)$

Attribute Values	Results	
	Method	B^*
$A_1 = (0, 5, 6)$	KH	$(6.36, 5.38, 7.38)$
$A_2 = (11, 13, 14)$	HCL	$(6.36, 6.58, 7.38)$
$B_1 = (0, 2, 4)$	HS	$(5.83, 6.26, 7.31)$
$B_2 = (10, 11, 13)$		

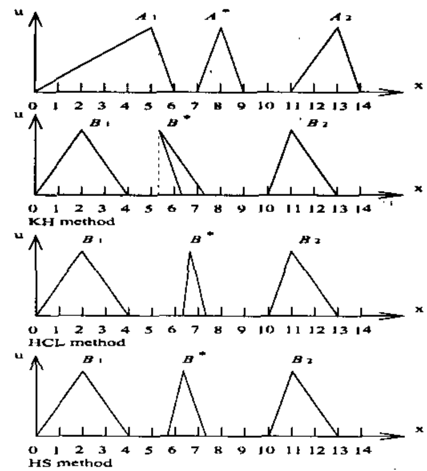

Fig. 6. The Reasoning Results of Example 1

KH method resulted in a nonconvex conclusion while the other two concluded with normal and convex fuzzy sets.

Example 2. The second case considers when the scale rate is ∞. The given observation is a fuzzy set $(5, 6, 8)$. Table II and Fig. 7 present the antecedents and interpreted fuzzy sets. The interpolation $(5.71, 6.28, 8.16)$ is obtained as follows: First the ratio between *COG level width* of A^* and the distance of $COG(A_1)$ and $COG(A_2)$ was calculated, then the *COG level width* of B^* was computed by retaining the same ratio but based on the distance of $COG(B_1)$ and $COG(B_2)$, and finally, the move transformation was applied as usual. The comparative results show that the KH and HCL methods performed similarly (the supports of the resultant fuzzy sets are identical since they are computed in the same way) while the HS method also generated a quite reasonable outcome.

Example 3. The third case considers a similar situation to example 1 but the observation is a singleton $A^* = (8, 8, 8)$.

TABLE II

RESULTS FOR EXAMPLE 2, WITH $A^* = (5, 6, 8)$

Attribute Values	Results	
	Method	B^*
$A_1 = (3, 3, 3)$	KH	$(5.33, 6.33, 9.00)$
$A_2 = (12, 12, 12)$	HCL	$(5.33, 6{:}55, 9.00)$
$B_1 = (4, 4, 4)$	HS	$(5.71, 6.28, 8.16)$
$B_2 = (10, 11, 13)$		

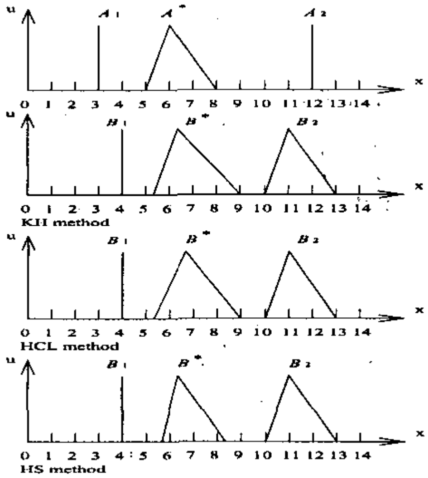

Fig. 7. The Reasoning Results of Example 2

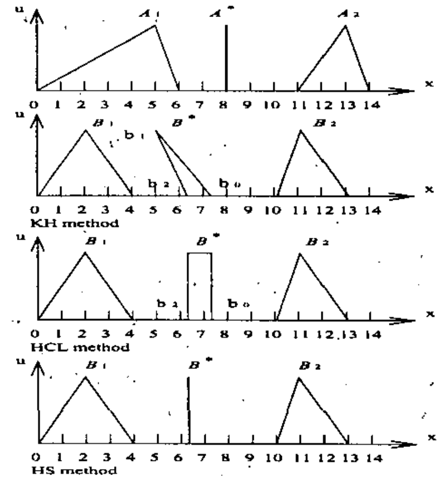

Fig. 8. The Reasoning Results of Example 3

TABLE III

RESULTS FOR EXAMPLE 3, WITH $A^* = (8, 8, 8)$

Attribute Values	Results	
	Method	B^*
$A_1 = (0, 5, 6)$		
$A_2 = (11, 13, 14)$	KH	(7.27, 5.38, 6.25)
$B_1 = (0, 2, 4)$	HCL	[7.27,6.25]
$B_2 = (10, 11, 13)$	HS	(6.49, 6.49, 6.49)

Table III and Fig. 8 present the results. In this case, the KH method once again generated a nonconvex fuzzy set and the HCL method produced a non-triangular fuzzy set. However, the method proposed in this paper resulted in a singleton conclusion, which is rather intuitive given the singleton-valued condition.

V. CONCLUSIONS

This paper has proposed a novel method for interpolative reasoning, based on the exploitation of the centre of gravity (COG) property of the fuzzy sets employed in fuzzy modelling. The method works by first constructing a new inference rule via manipulating two adjacent rules (and the given observations of course), and then by using similarity information to convert the intermediate inference results into the final derived conclusion. To support this, two transformation operations have been introduced, which allow the COG of a fuzzy set to remain unaltered before and after the transformation. This approach not only inherits the common advantages of fuzzy interpolative reasoning: allowing inferences to be performed with simple and sparse rule bases, but also guarantees that the resultant fuzzy values of an inference remain to be normal and convex. This helps maintain the desirable practical property of fuzzy systems in that their modelling and inference are easily interpretable.

Much can be improved, however. In particular, the present work only uses triangular fuzzy sets in fuzzy rules. Other types of fuzzy set representation (e.g., trapezoidal and bell-shaped) are also often utilised in fuzzy modelling. An extension of the proposed method to cope with such more complex representations is worth investigating. In addition, this work does not look into the possible effect of arranging the rule base in a certain partial order for rules of complex condition patterns. Further effort to estimate the overheads this may cause over the inference procedures seems necessary.

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